Our Solar System in the Layout at Giza

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Contents

1 Purpose

This paper demonstrates a procedure for checking coincidence of arbitrary points with planetary orbits, followed by an application to the layout of the great pyramids at Giza.

2 Procedure

The steps used to implement the procedure are as follows:

2.1 Overview

- Develop equations regarding generalized points representing orbital centers
- Develop normalized model and equations for scaling
- Examine Giza topography and develop model and grid
- Determine circle of suitable orbital centers for Earth and Venus
- Determine circles of suitable orbital centers for Earth and Mercury
- Determine circles of suitable orbital centers for Earth and Mars
- Check intersection of circles to check coincidence with mutual orbits

2.2 Two Points

Consider two points e and v separated by a distance V. For convenience place point e at the origin and point v at a distance V along the x-axis as shown in Figure 1.

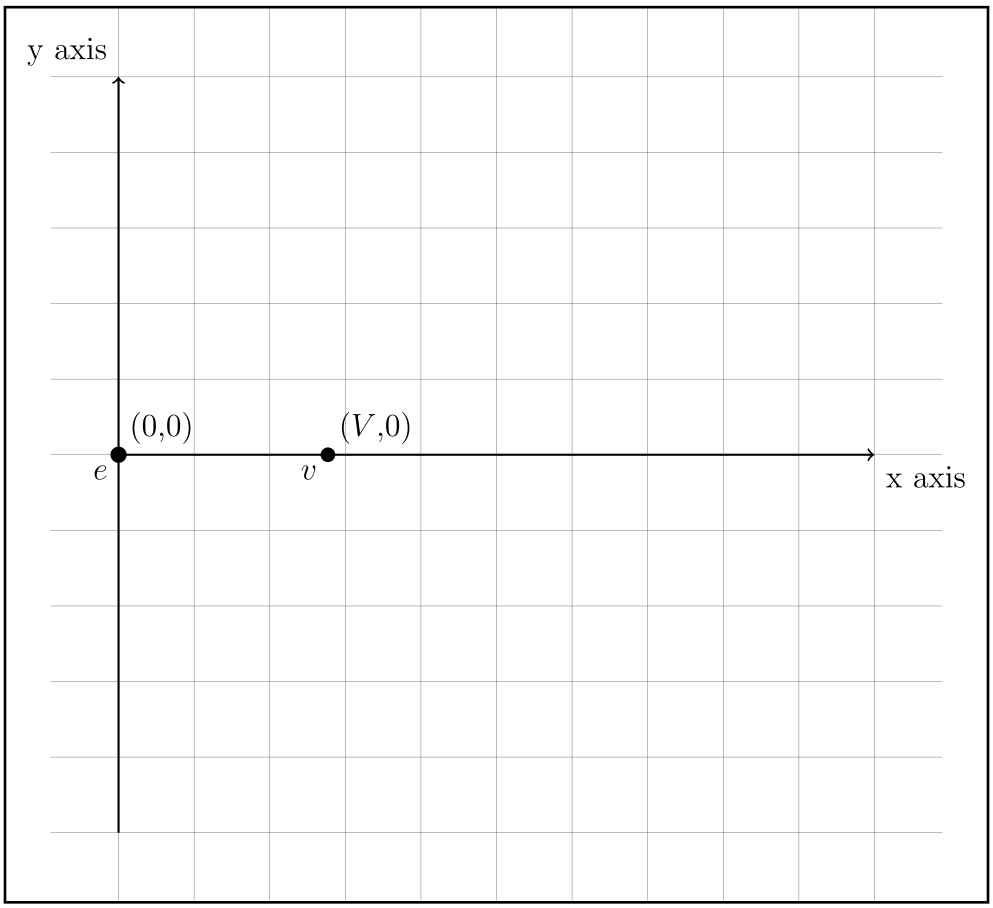

Figure 1:
Two Points

2.3 Two Points and a Third Point

Next consider a third point s anywhere on the grid as shown in Figure 2. s is distance r_e from e, and distance r_v from v.

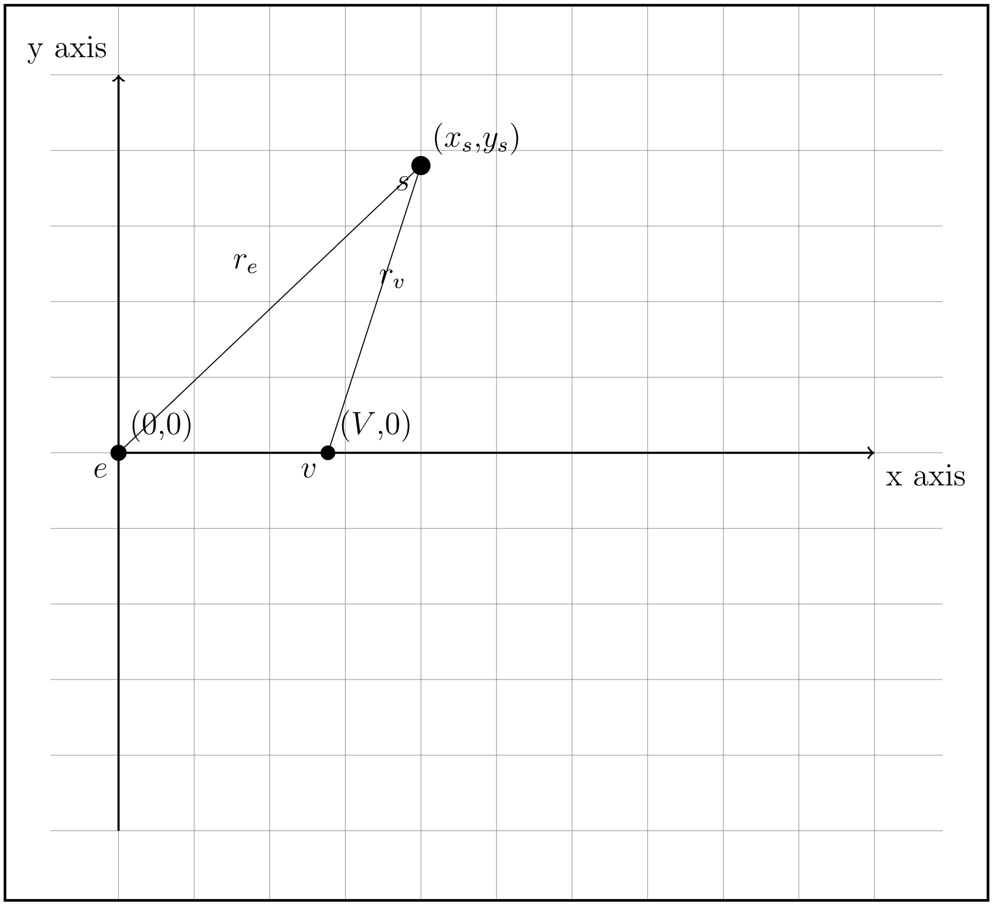

Figure 2:
Two Points and a Third Point

Figure 3 interprets r_e and r_v as the radii of two circles with e and v on their respective circumferences and s at their centers.

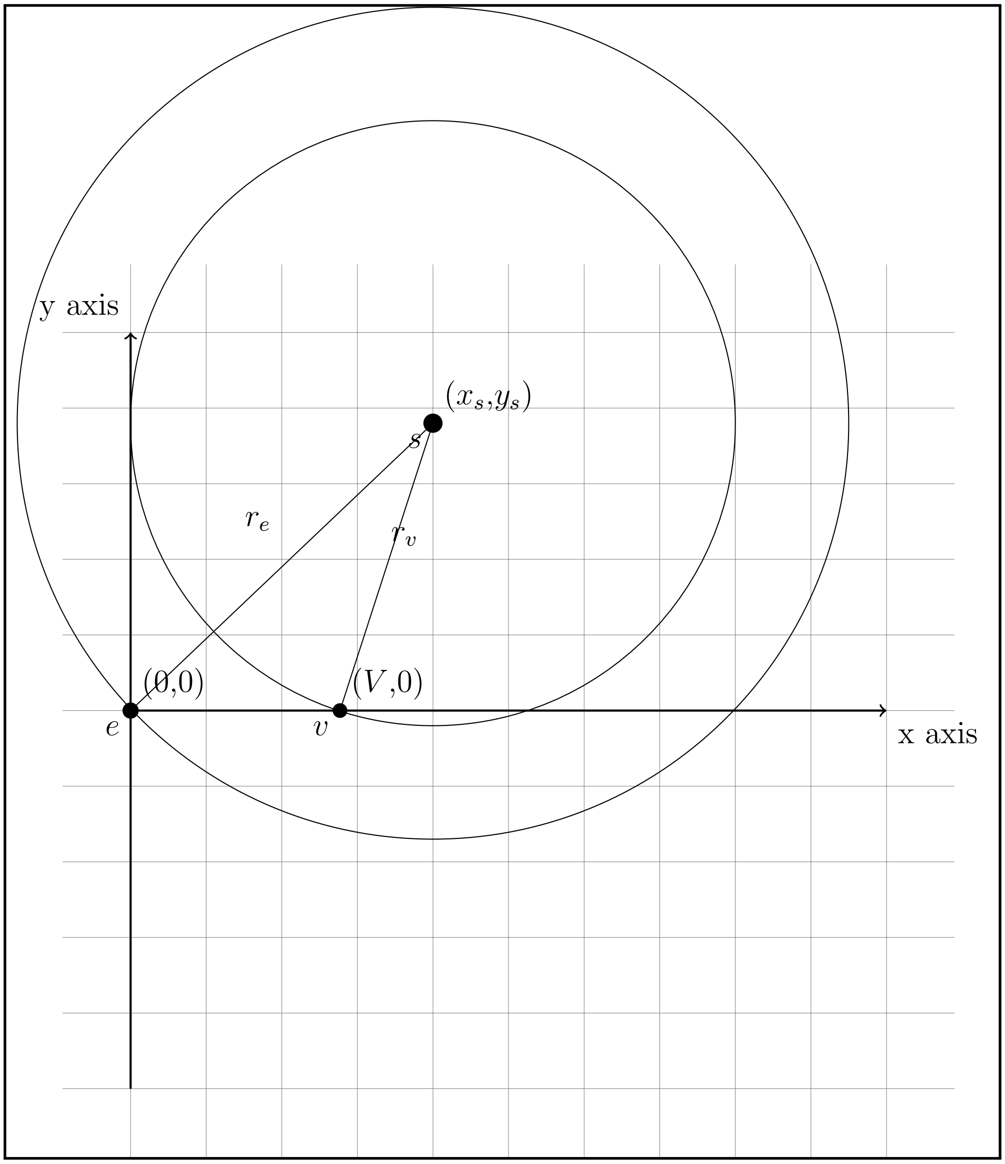

Figure 3:
Two Points Orbiting a Third Point

2.4 Euclidean Distances

The Euclidean distances r_e and r_v can be calculated using the Cartesian coordinates of the points.

$$r_e^2 = (x_s - x_e)^2 + (y_s - y_e)^2 \tag{1}$$

$$r_v^2 = (x_s - x_v)^2 + (y_s - y_v)^2 \tag{2}$$

Since e is at the origin, $x_e = y_e = 0$.
Since v is on the x-axis, x_v =V and $y_v = 0$.
The distance equations simplify to:

$$r_e^2 = x_s^2 + y_s^2 \tag{3}$$

$$r_v^2 = (x_s - V)^2 + y_s^2 \tag{4}$$

Define k as the fraction of these lengths:

$$k = \frac{r_v}{r_e} \tag{5}$$

That is, k is the ratio of the shorter length to the longer length.

2.5 Distance Equations

The distance equations equate a proportional length of r_e to r_v.

$$kr_e = r_v \tag{6}$$

$$k\sqrt{x_s^2 + x_s^2} = \sqrt{(x_s - V)^2 + y_s^2} \tag{7}$$

$$k^2(x_s^2 + y_s^2) = (x_s - V)^2 + y_s^2 \tag{8}$$

$$k^2x_s^2 + k^2y_s^2 = (x_s - V)^2 + y_s^2 \tag{9}$$

$$k^2y_s^2 = -k^2x_s^2 + (x_s - V)^2 + y_s^2 \tag{10}$$

$$k^2y_s^2 - y_s^2 = -k^2x_s^2 + (x_s - V)^2 \tag{11}$$

$$(k^2 - 1)y_s^2 = -k^2x_s^2 + (x_s - V)^2 \tag{12}$$

$$(k^2 - 1)y_s^2 = -k^2x_s^2 + x_s^2 - 2Vx_s + V^2 \tag{13}$$

$$(k^2 - 1)y_s^2 = -(k^2 - 1)x_s^2 - 2Vx_s + V^2 \tag{14}$$

$$-(1 - k^2)y_s^2 = +(1 - k^2)x_s^2 - 2Vx_s + V^2 \tag{15}$$

$$(1 - k^2)y_s^2 = -(1 - k^2)x_s^2 + 2Vx_s - V^2 \tag{16}$$

$$y_s^2 = -x_s^2 + \frac{2V}{(1 - k^2)}x_s - \frac{V^2}{(1 - k^2)} \tag{17}$$

$$x_s^2 + y_s^2 = \frac{2V}{(1 - k^2)}x_s - \frac{V^2}{(1 - k^2)} \tag{18}$$

$$x_s^2 + y_s^2 = (\frac{2V}{(1 - k^2)})(x_s - \frac{V}{2}) \tag{19}$$

2.6 Apollonian Circles

Typically a circle is defined as a set of points at a constant distance (the radius r) from another point (the center c). A circle can also be defined as a set of points that represent a constant ratio between two fixed points. Such a circle is referred to as an Apollonian Circle. The Apollonian Circle for points e and v with ratio k is shown in Figure 4.

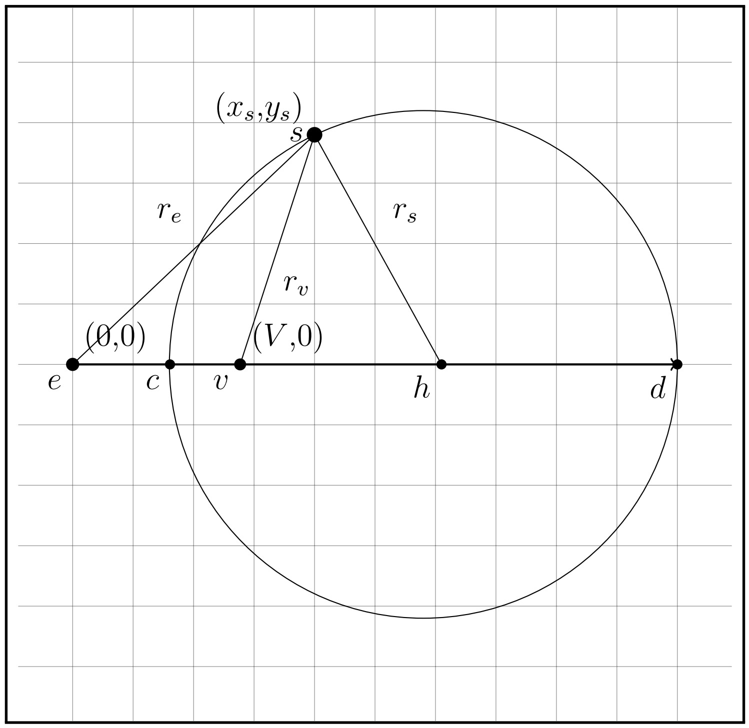

Figure 4:
Apollonian Circle Variables

2.6.1 Apollonian Circle Variables

Consider the two points on this circle that intersect the x-axis ($y_s = 0$). These are points c and d. Point h is defined as exactly half-way between them.

2.6.2 Maximum Intercept - x_d

Point d at $x_d = (D, 0)$ represents the maximum value for r_e and r_v.

$$V = D * (1 - k) \tag{20}$$

where $k < 1$.

2.6.3 Normalized System

The system can be normalized by defining the largest magnitude in x as 1, that is set point x_d at (1,0) by scaling all values by D. After scaling:

$$k = 1 - V \tag{21}$$

$$k^2(x_s^2 + y_s^2) = (x_s - (1 - k))^2 + y_s^2 \tag{22}$$

2.6.4 Minimum Intercept - x_c

Consider the point on the x-axis at which these conditions are true at x_c. This point represents the minimum value for r_e and r_v. At this point c ($y_c = 0$):

$$\frac{V - x_c}{x_c} = k \tag{23}$$

$$k = \frac{V - x_c}{x_c} \tag{24}$$

$$k = \frac{(1 - k) - x_c}{x_c} \tag{25}$$

$$kx_c = (1 - k) - x_c \tag{26}$$

$$kx_c + x_c = (1 - k) \tag{27}$$

$$(k + 1)x_c = (1 - k) \tag{28}$$

$$x_c = \frac{(1 - k)}{(1 + k)} \tag{29}$$

Subtracting the two values gives the diameter of the circle, twice the radius.

$$r_s = \frac{1 - x_c}{2} \tag{30}$$

2.6.5 Center of Apollonian Circle - x_h

The center, point h, is at the maximum value subtracting the radius, or the lower value adding it.

$$x_h = \frac{1}{2}(x_d + x_c) \tag{31}$$

$$x_h = \frac{1}{2}(1 + \frac{(1-k)}{(1+k)}) \tag{32}$$

$$x_h = \frac{1}{2}(\frac{(1+k+1-k)}{(1+k)}) \tag{33}$$

$$x_h = \frac{1}{(1+k)} \tag{34}$$

The midpoint can also be found using calculus when y-value changes slope in equation (17):

$$y^2 = -x^2 + \frac{2(1-k)}{(1-k^2)}x - \frac{(1-k)^2}{(1-k^2)} \tag{35}$$

Take derivative of both sides by dx:

$$2y\frac{dy}{dx} = -2x + \frac{2(1-k)}{1-k^2} \tag{36}$$

at x_h where $\frac{dy}{dx} = 0$, left side of equation is 0.

$$2x_h = \frac{2(1-k)}{1-k^2} \tag{37}$$

$$x_h = \frac{1-k}{1-k^2} \tag{38}$$

Check against earlier value

$$\frac{1-k}{1-k^2} \stackrel{?}{=} \frac{1}{(1+k)} \tag{39}$$

$$(1-k)(1+k) \stackrel{?}{=} 1-k^2 \tag{40}$$

$$1-k^2 \stackrel{!}{=} 1-k^2 \tag{41}$$

3 Planets

3.1 Kepler's Variables

Kepler's Laws characterize planetary orbits according to mathematical features. Among these features are the Semi-Major Axis (SMa) and the Eccentricity (Ecc) of each planet's orbit. To the first order an orbit can be represented by a circle, each world traveling a path of points at the same distance around the sun. This distance is the SMa and the extent to which the circle becomes increasingly elliptical is represented by the Ecc.

3.2 Solar System model

In the model points e and v represent two different planets with different SMa. Point s represents the sun, the orbital center of the planets. Points e and v are located on concentric circles around s with radii r_e and r_v as shown in Figure 3.

3.3 Our Solar System

The inner planets of our solar system can be modeled as a set of consistently scaled concentric circles with the sun at the origin. Orbital extremes are color coded blue for outer, red for inner, and green for SMa. This model is shown in Figure 5.

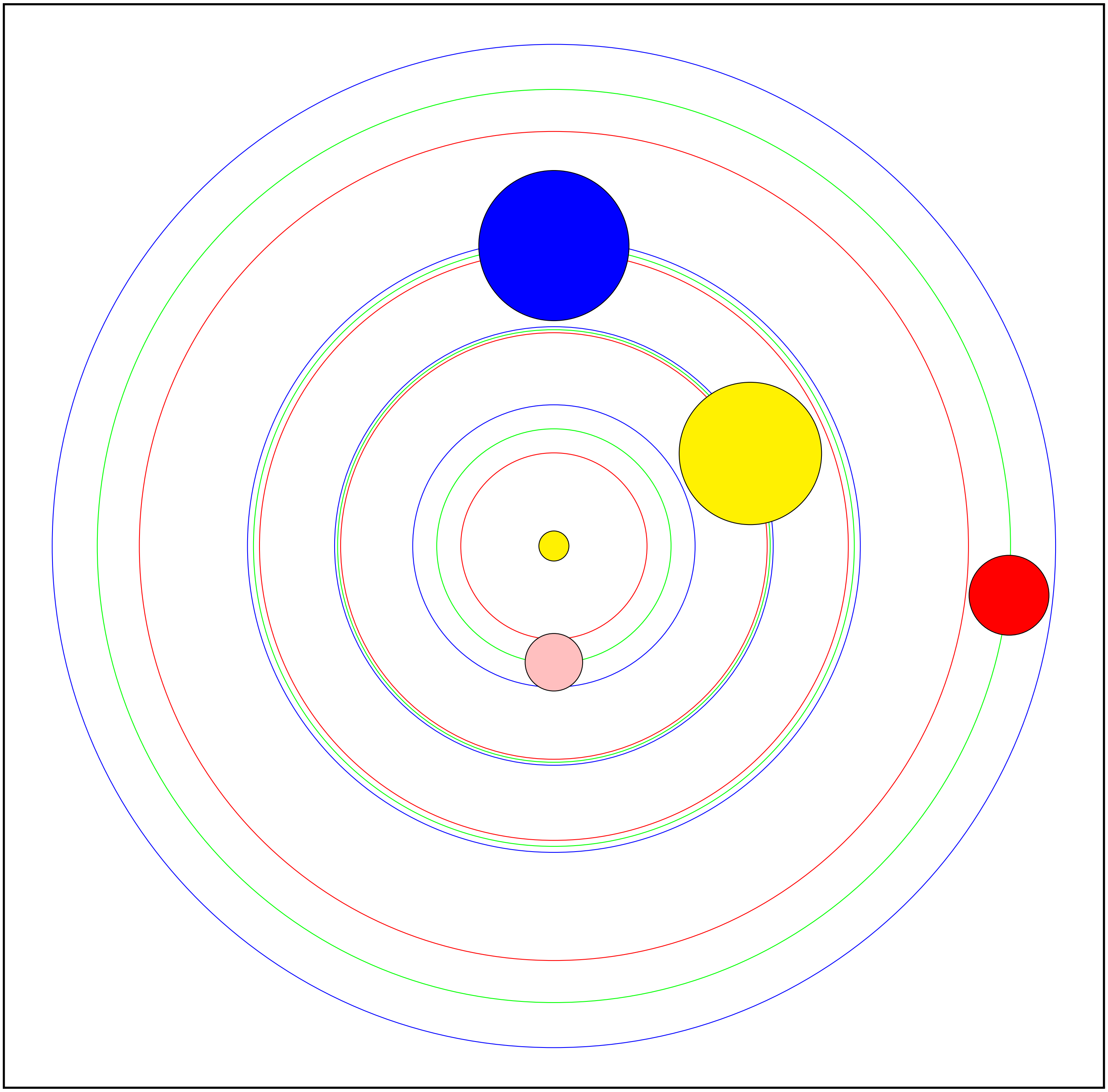

Figure 5:
Inner solar system. Scale of orbits does not equal scale of objects.

3.4 Astronomical Unit Scaling

In the normalized system, we can take advantage of solar system scaling using the Astronomical Unit (A.U.). The distance from Earth to the Sun is 1 A.U.

3.5 Calculating with the Normalized System

The normalized system is considered from the outer point at the origin point e. That is, the outermost planet is at point e and the innermost planet is at point v. The sun is at point s, at coordinates (1,0). Therefore when considering Earth and Venus or Earth and Mercury, Earth will be at the origin. When considering Earth and Mars, then Mars will be at the origin and Earth at point v. All results must be scaled and rescaled accordingly.

3.6 Apollonian Circles for Earth-Venus

Venus varies between 0.718 and 0.728 A.U. with $SMa = 0.723$.

3.6.1 SMa

Substituting $k = 0.723$ in equations (29), (30), and (34):

$$x_c = \frac{(1 - 0.723)}{(1 + 0.723)} = 0.161 \tag{42}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.723} = 0.580 \tag{43}$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.161}{2} = 0.420 \tag{44}$$

A circle with radius=0.420 located at (0.580,0) is shown in Figure 6

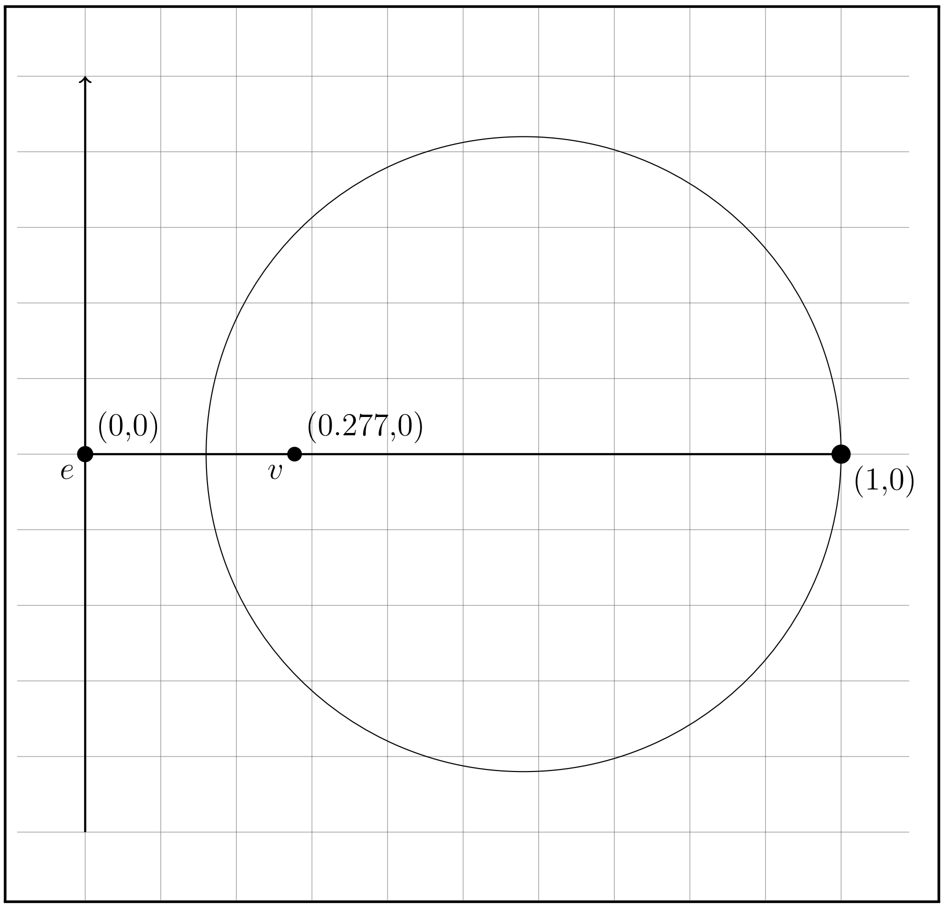

Figure 6:
Normalized Apollonian Circle for Earth Venus SMa

3.6.2 Outer

$k = 0.728$

$$x_c = \frac{(1 - 0.728)}{(1 + 0.728)} = 0.157 \tag{45}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.723} = 0.579 \tag{46}$$

$$r_{ac} = \frac{1 - x_c}{2} = \frac{1 - 0.157}{2} = 0.421 \tag{47}$$

A circle with radius=0.421 located at (0.579,0) is shown in Figure 7.

3.6.3 Inner

$k = 0.718$

$$x_c = \frac{(1 - 0.718)}{(1 + 0.718)} = 0.164 \tag{48}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.723} = 0.582 \tag{49}$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.164}{2} = 0.418 \tag{50}$$

A circle with radius=0.418 located at (0.582,0) is shown in Figure 7.

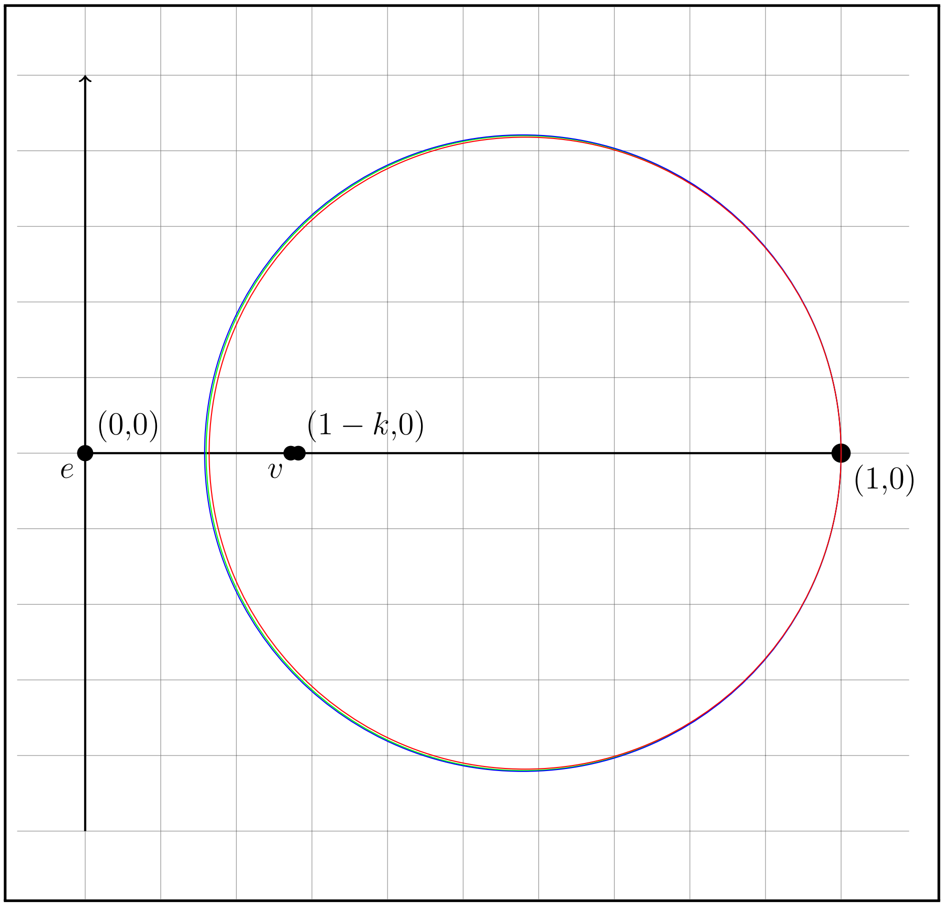

Figure 7:
Normalized Apollonian Circles Earth Venus Outer Inner

Due to the low eccentricity of Venus, for these purposes only the SMa Apollonian Circle will be used.

3.7 Apollonian Circles for Earth-Mercury

Mercury varies between 0.307 and 0.467 A.U. with $SMa = 0.387$.

3.7.1 SMa

$k = 0.387$

$$x_c = \frac{(1-0.387)}{(1+0.387)} = 0.442 \tag{51}$$

$$x_h = \frac{1}{1+k} = \frac{1}{1.387} = 0.721 \tag{52}$$

$$r_s = \frac{1-x_c}{2} = \frac{1-0.442}{2} = 0.279 \tag{53}$$

A circle with radius=0.279 located at (0.721,0) is shown in Figure 8.

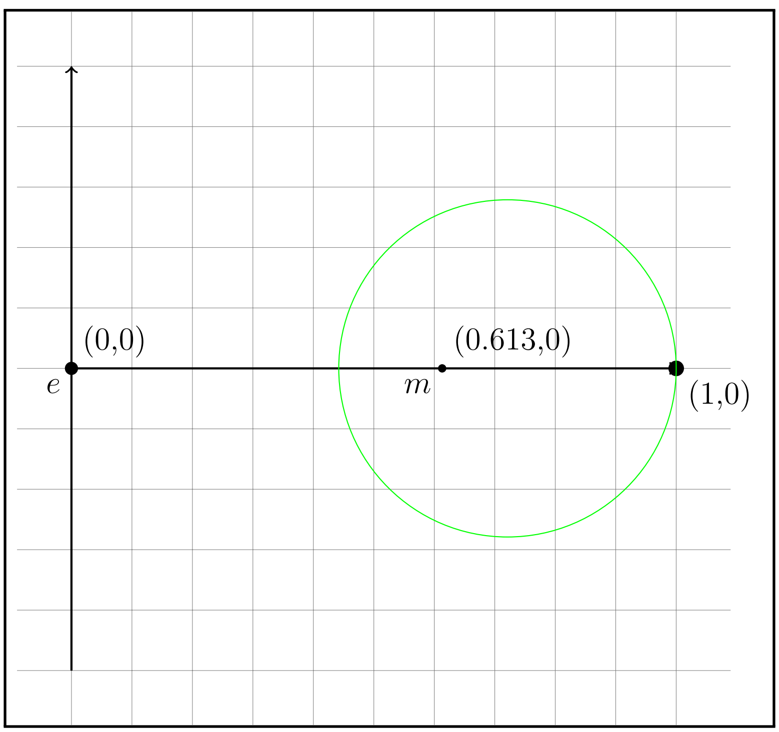

Figure 8:
Normalized Apollonian Circle Earth Mercury SMa

3.7.2 Outer

$k = 0.467$

$$x_c = \frac{(1 - 0.467)}{(1 + 0.467)} = 0.363 \tag{54}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.467} = 0.680 \tag{55}$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.363}{2} = 0.318 \tag{56}$$

A circle with radius=0.318 located at (0.680,0) is shown in Figure 9.

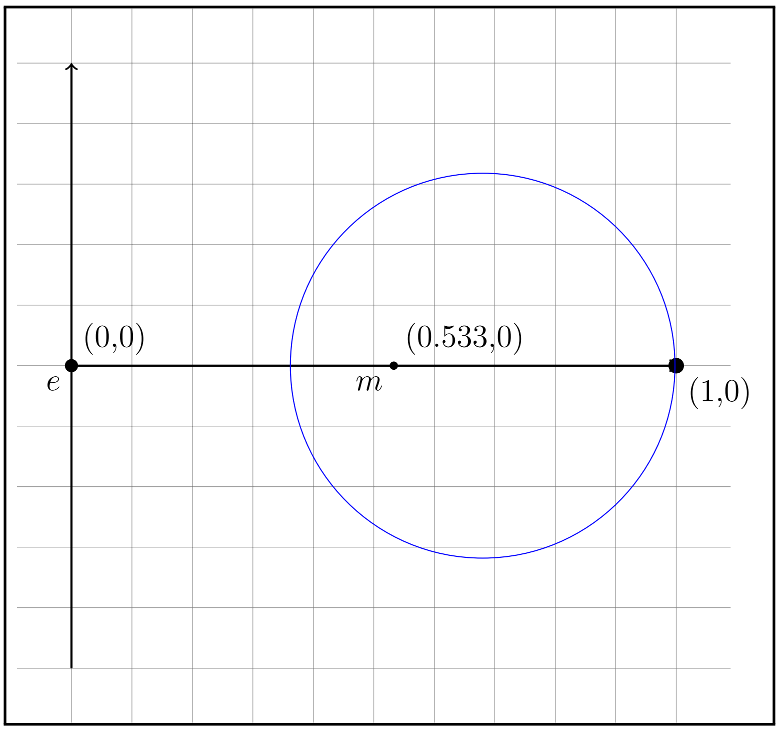

Figure 9:
Normalized Apollonian Circle Earth Mercury Outer

3.7.3 Inner

$k = 0.307$

$$x_c = \frac{(1-0.307)}{(1+0.307)} = 0.530 \tag{57}$$

$$x_h = \frac{1}{1+k} = \frac{1}{1.307} = 0.765 \tag{58}$$

$$r_s = \frac{1-x_c}{2} = \frac{1-0.530}{2} = 0.235 \tag{59}$$

A circle with radius=0.235 located at (0.765,0) is shown in Figure 10.

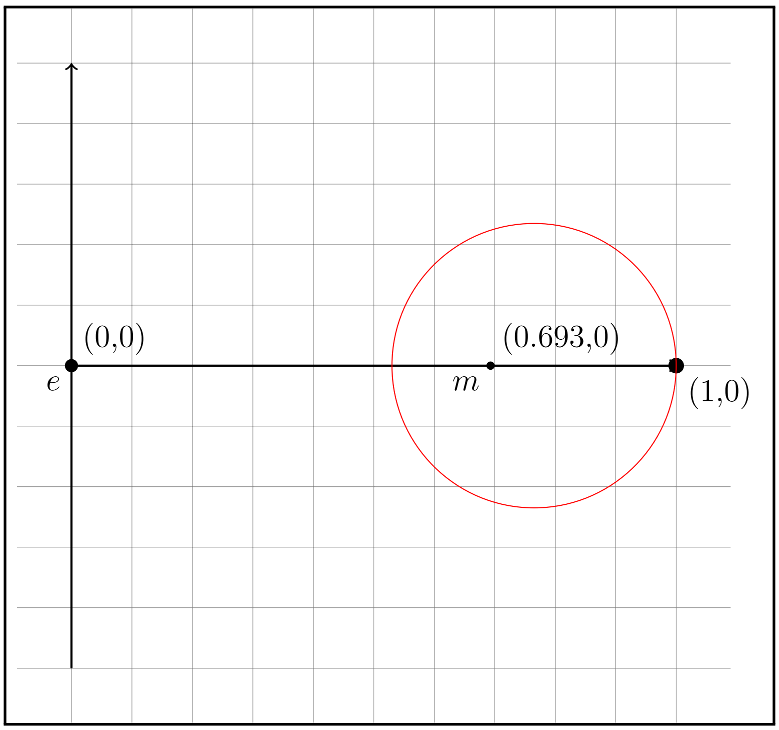

Figure 10:
Normalized Apollonian Circle Earth Mercury Outer

3.8 Apollonian Circles for Mars-Earth

Mars varies between 1.3814 and 1.6660 A.U. with $SMa = 1.5237$.

3.8.1 SMa

For Mars-Earth in the normalized system,

$$k = \frac{(1)}{1.5237} = 0.6563 \tag{60}$$

$$x_c = \frac{(1 - 0.6563)}{(1 + 0.6563)} = 0.2075 \tag{61}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.6563} = 0.6038 \tag{62}$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.2075}{2} = 0.3962 \tag{63}$$

A circle with radius=0.3962 located at (0.6038,0) is shown in Figure 11.

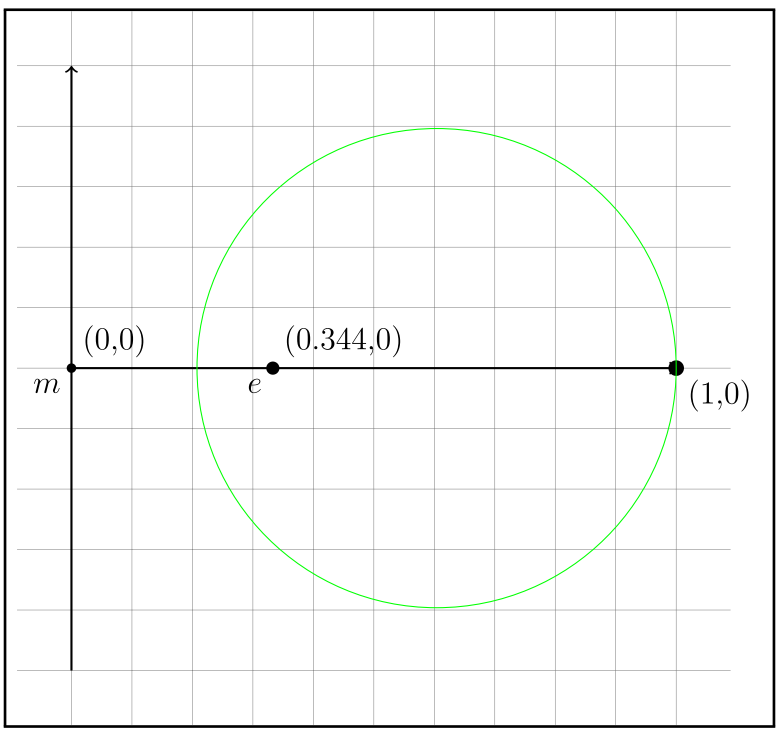

Figure 11:
Normalized Apollonian Circle Mars Earth SMa

3.8.2 Outer

$$k = \frac{(1)}{1.6660}) = 0.6002 \tag{64}$$

$$x_c = \frac{(1 - 0.6002)}{(1 + 0.6002)} = 0.2498 \tag{65}$$

$$x_h = \frac{1}{1 + k} = \frac{1}{1.6002} = 0.6249 \tag{66}$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.2498}{2} = 0.3751 \tag{67}$$

A circle with radius=0.3751 located at (0.6249,0) is shown in Figure 12.

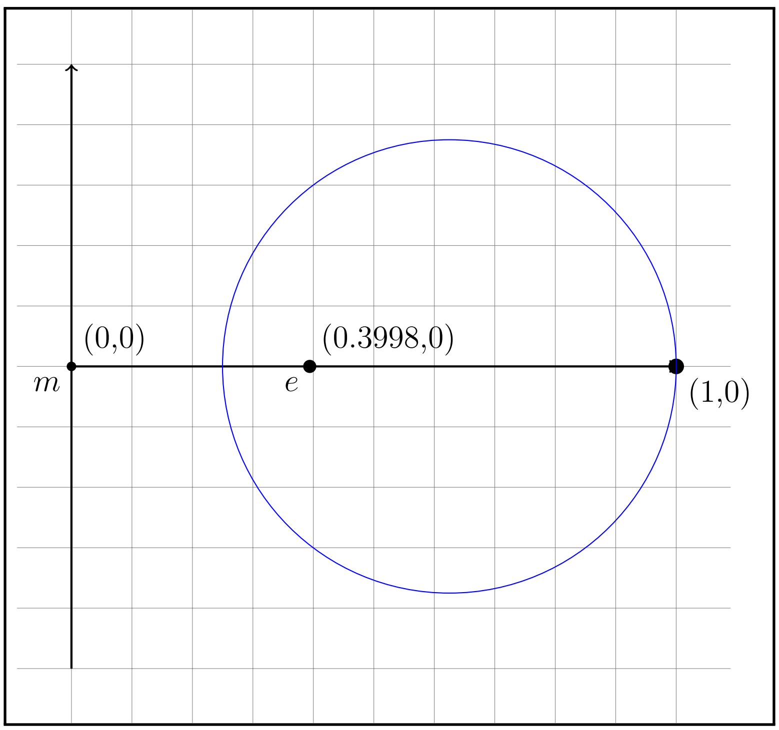

Figure 12:
Normalized Apollonian Circle Mars Earth Outer

3.8.3 Inner

$$k = \frac{(1)}{1.3814}) = 0.7239 \quad (68)$$

$$x_c = \frac{(1 - 0.7239)}{(1 + 0.7239)} = 0.1602 \quad (69)$$

$$x_h = \frac{1}{1+k} = \frac{1}{1.7239} = 0.5801 \quad (70)$$

$$r_s = \frac{1 - x_c}{2} = \frac{1 - 0.1602}{2} = 0.4199 \quad (71)$$

A circle with radius=0.4199) located at (0.5801,0) is shown in Figure 13.

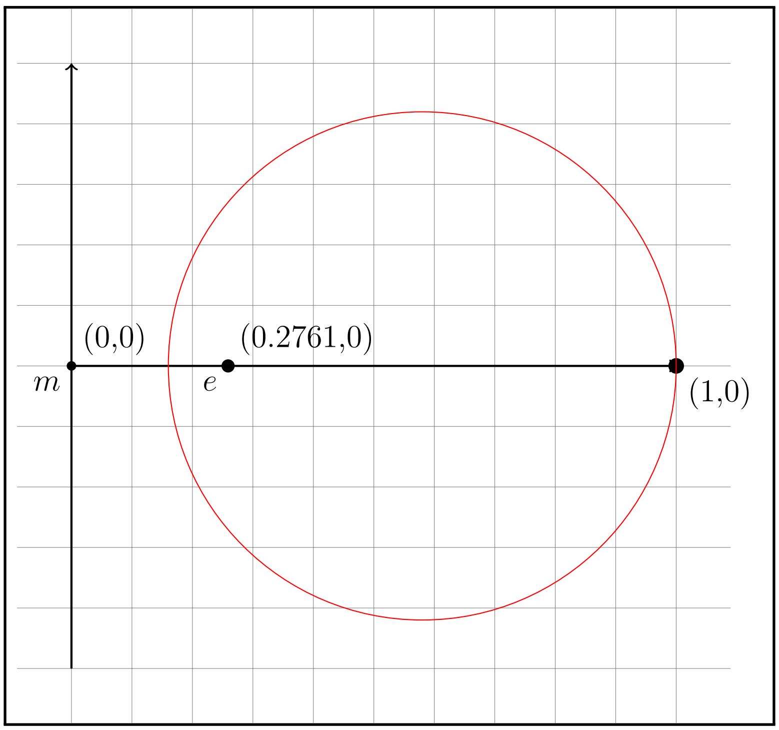

Figure 13:
Normalized Apollonian Circle Mars Earth Inner

3.9 Applying the math

This normalized system can be rotated and uniformly scaled around the origin. Such operations can be used to scale and orient the results to the size of the Giza layout.

4 Pyramids

Global coordinates accessed from Wikipedia.

4.1 Earth Pyramid

$29^\circ\ 58'\ 45.03''$ N (+29.979175)
$31^\circ\ 8'\ 3.69''$ E (+31.134358)

4.2 Venus Pyramid

$29^\circ\ 58'\ 34''$ N (+29.976111)
$31^\circ\ 7'\ 51''$ E (+31.130833)

4.3 Mars/Mercury Pyramid

$29^\circ\ 58'\ 21''$ N (+29.9725)
$31^\circ\ 7'\ 42''$ E (+31.128333)

4.4 Royal Cubit

Conversion between meter and royal cubit (rc)

$$1rc = \frac{\pi}{6} meters \tag{72}$$

4.5 Grid

The grid is scaled by the Earth pyramid using its size as a reference unit (440rc×440rc). From that reference unit extends a four by four grid. Each side of this grid is $1760rc$ long. This grid is shown in Figure 14.

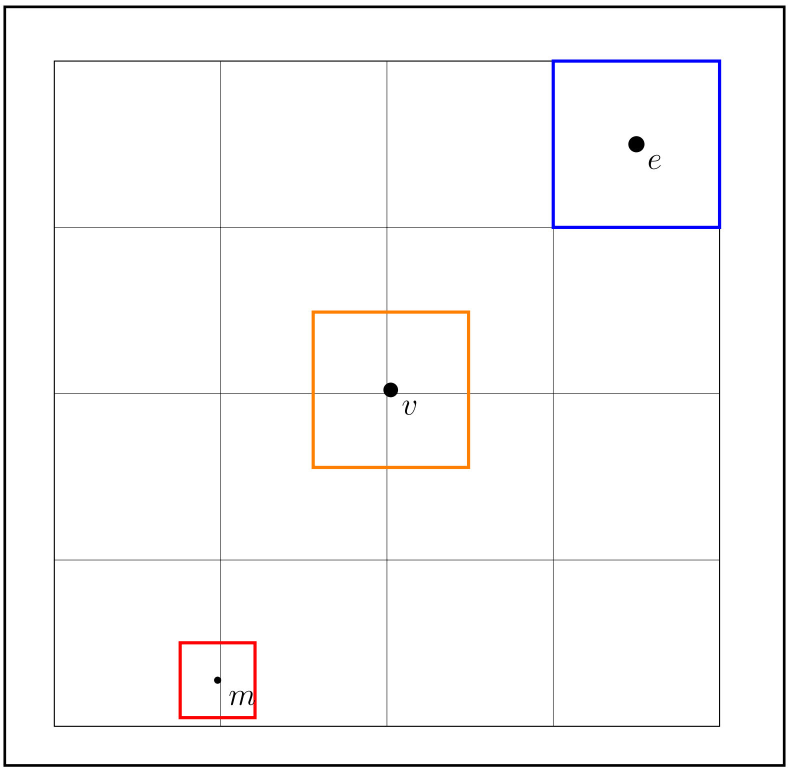

Figure 14:
Grid with Pyramids

4.6 Haversine Formula

The Haversine Formula cam be used to compute the shortest distance over the Earths surface between two points. For this article the online calculator at http://www.movable-type.co.uk/scripts/latlong.html was applied to the coordinate data.

4.7 Earth Pyramid to Venus Pyramid

Referenced from the Great Pyramid, the Venus pyramid is located 481.0m away at a bearing of $224^{\circ}\ 54'$. This is almost directly due south west.

4.8 Earth Pyramid to Mercury/Mars Pyramid

Referenced from the Great Pyramid, the third pyramid is located 942.2m away at a bearing of $218^{\circ}\ 01'$.

4.9 Reference Distances

These reference distances and bearings are shown in Figure 15.

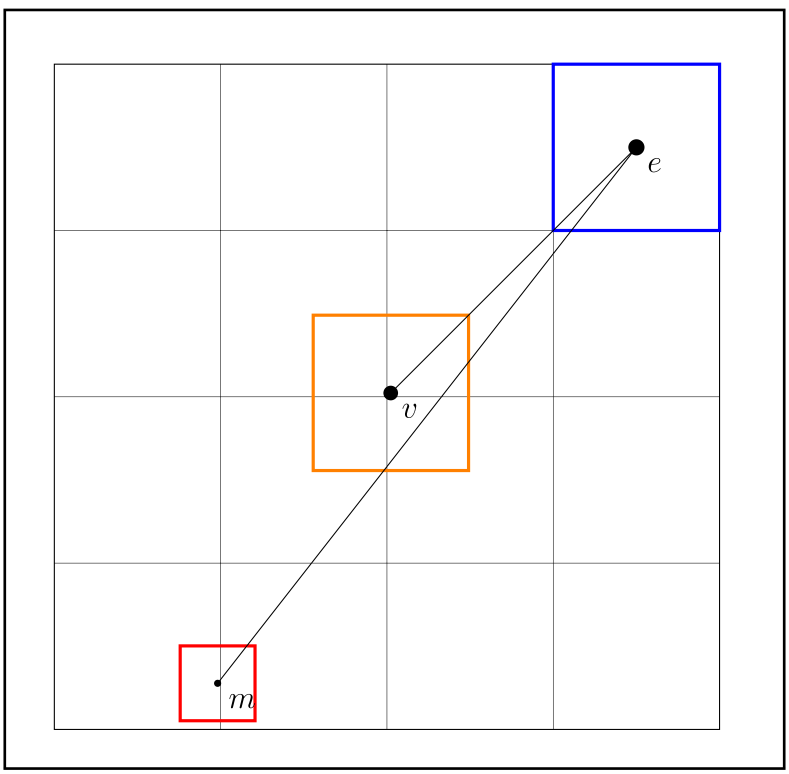

Figure 15:
Grid with Reference Distances

5 Calculations

5.1 Overview

The basic element of the grid is scaled by the Earth pyramid. It is the reference point, the origin, for all calculations unless otherwise noted.

5.2 Scaling the Normalized Model

In order to translate these results from the normalized model to actual physical coordinates,

$$V = D(1 - k) \tag{73}$$

where V and D are actual physical distances.

5.3 Scaling Earth-Venus

For scaling Earth-Venus, the distance V on the ground between the larger pyramids is 481.0m and $k = 0.723$.

$$D = \frac{V}{1-k} = \frac{481.0}{1-0.723} = 1736m \tag{74}$$

Results calculated in section 3.5 should be scaled by this value.

$$x_c * D = 0.1608 * 1736m = 279.1m \tag{75}$$

$$x_h * D = 0.5804 * 1736m = 1008m \tag{76}$$

$$r_s = \frac{D - x_c * D}{2} = \frac{1736m - 279.1m}{2} = 728.5m \tag{77}$$

This circle (radius= $728.5m$) located 1008 meters southwest of the Great Pyramid is shown in Figure 16.

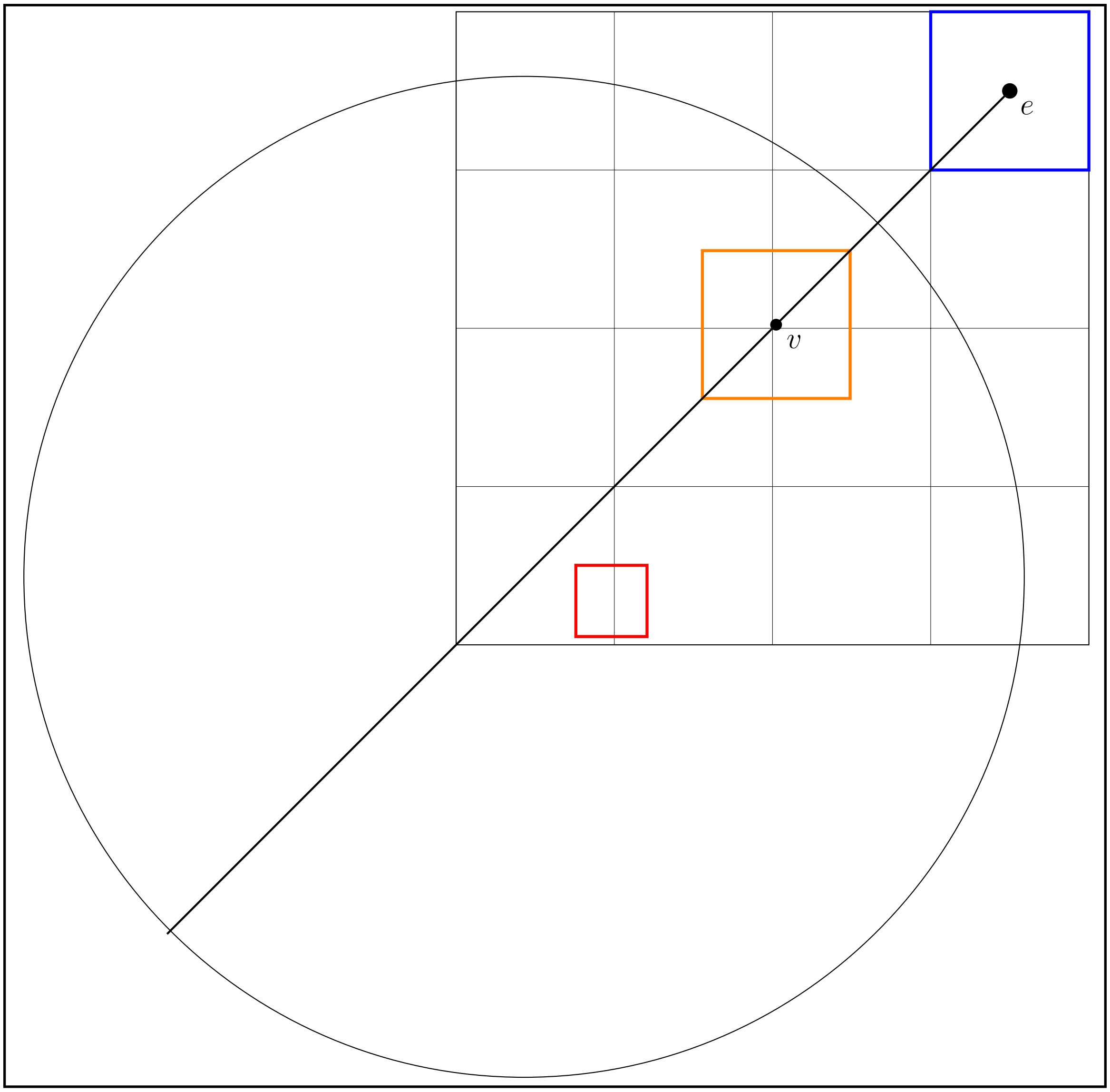

Figure 16:
Apollonian Circle of Earth Venus

5.4 Scaling Earth-Mercury

The eccentricity of Mercury must be accounted for, generating three Apollonian Circles representing the average and extremes of its orbit. First the SMA (average) value is calculated then the outer (maximum) and inner (minimum) values.)) For Earth-Mercury, distance V on the ground between the largest and smallest pyramids is 942.2m.

5.4.1 Mercury SMa

$$k = 0.387 \tag{78}$$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.387} = 1537m \tag{79}$$

Results calculated in section 3.6 should be scaled by this value.

$$x_c * D = 0.442 * 1537m = 679m \tag{80}$$

$$x_h * D = 0.721 * 1537m = 1108m \tag{81}$$

$$r_s = \frac{1537m - 679m}{2} = 429m \tag{82}$$

5.4.2 Mercury Outer

$$k = 0.467 \tag{83}$$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.467} = 1768m \tag{84}$$

All values from the normalized Apollonian Circle should be scaled by this value for the outer case.

$$X_c = 0.363 * 1768m = 642m \tag{85}$$

$$X_h = 0.680 * 1768m = 1202m \tag{86}$$

$$r_{ac} = \frac{1768m - 642m}{2} = 563m \tag{87}$$

5.4.3 Mercury Inner

$$k = 0.307 \tag{88}$$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.307} = 1360m \tag{89}$$

All values from the normalized Apollonian Circle should be scaled by this value for the inner case.

$$x_c * D = 0.530 * 1360m = 720.8m \tag{90}$$

$$x_h * D = 0.765 * 1360m = 1040.4m \tag{91}$$

$$r_s = \frac{1360m - 720.8m}{2} = 319.6m \tag{92}$$

These three circles are shown in Figure 17.

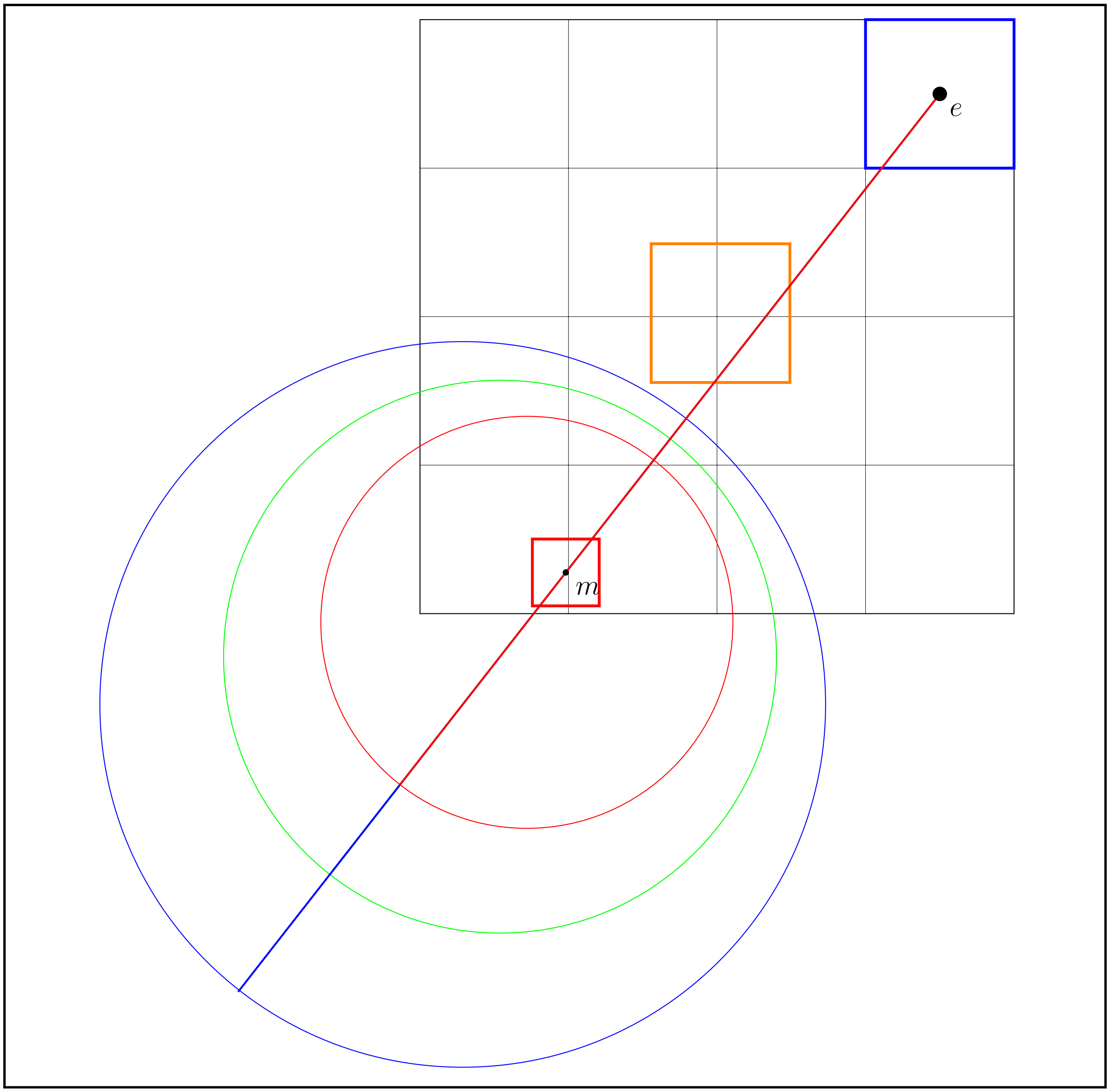

Figure 17:
Apollonian Circles of Earth Mercury

5.5 Scaling Mars-Earth

As in the case with Mercury, three Apollonian Circles will be generated representing the extremes of the orbit of Mars. The distance V is again 942.2m.

5.5.1 Mars SMa

$k = .6563$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.6563} = 2741.34m \quad (93)$$

Results calculated in section 3.7 should be scaled by this value.

$$x_c * D = 0.2075 * 2741.34m = 568.828m \quad (94)$$

$$x_h * D = 0.6038 * 2741.34m = 1655.2211m \quad (95)$$

$$r_s = \frac{2741.34m - 568.828m}{2} = 1086.256m \quad (96)$$

5.5.2 Mars Outer

$$k = .6002 \quad (97)$$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.6002} = 2356.68m \quad (98)$$

All values from the normalized Apollonian Circle should be scaled by this value for the SMa case.

$$x_c * D = 0.2498 * 2356.68m = 588.698m \quad (99)$$

$$x_h * D = 0.6249 * 2356.68m = 1472.6893m \quad (100)$$

$$r_s = \frac{2356.68m - 588.698m}{2} = 883.99m \quad (101)$$

5.5.3 Mars Inner

$$k = .7239 \quad (102)$$

$$D = \frac{V}{1-k} = \frac{942.2}{1-0.7239} = 3412.532m \quad (103)$$

All values from the normalized Apollonian Circle should be scaled by this value for the SMa case.

$$x_c * D = 0.1602 * 3412.532m = 546.688m \quad (104)$$

$$x_h * D = 0.5801 * 3412.532m = 1979.6096m \quad (105)$$

$$r_s = \frac{3412.532m - 546.688m}{2} = 1432.922m \quad (106)$$

These three circles are shown in Figure 18.

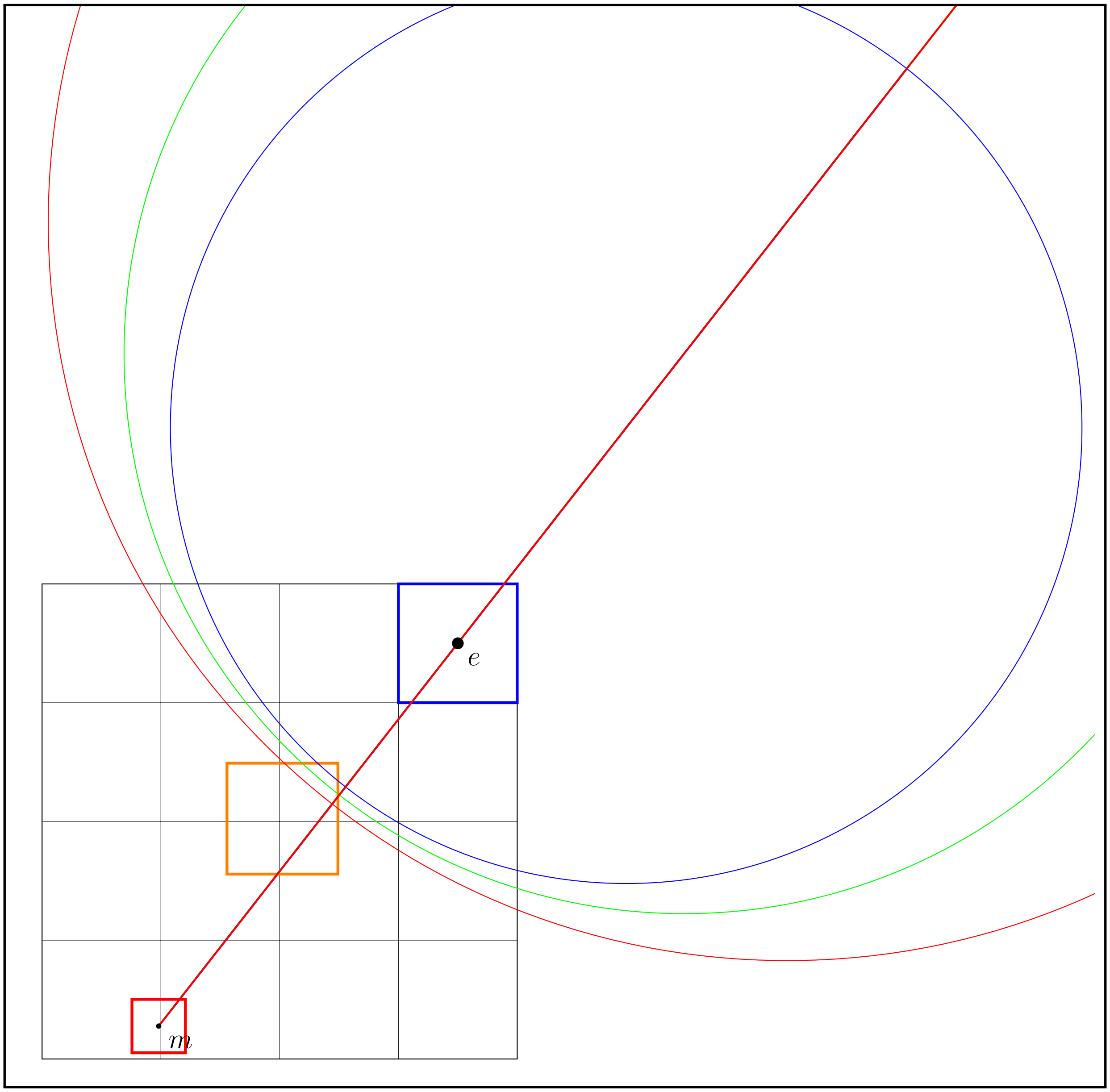

Figure 18:
Apollonian Circle of Mars Earth

6 Analysis

6.1 Intersecting Circles

This procedure uses scale invariance, that is all bodies are scaled the same relative to each other. Knowing in general if:

A is to B
A is to C
then B is to C

then specifically regarding the orbital ratios of SMa values found in this problem:

E is to V
E is to M
then V is to M

By constructing mutual Apollonian Circles at the same scale, planetary layouts will be sensible only at intersection points between these Circles. If any point on the Apollonian Circle representing Earth-Venus also intersects the Apollonian Circle representing Mercury or Mars, then at that point the value of k for both Circles is the same. The model is to scale at that point.

6.2 Earth Venus Mercury

Figure 19 shows the Apollonian Circles of Earth-Venus (Figure 16) and Earth-Mercury (Figure 17). Of the three Earth-Mercury circles only the outer intersects the Apollonian Circle of Earth-Venus. The model of the solar system does not hold at SMa or the inner extreme of Mercury's orbit since there is no intersection.

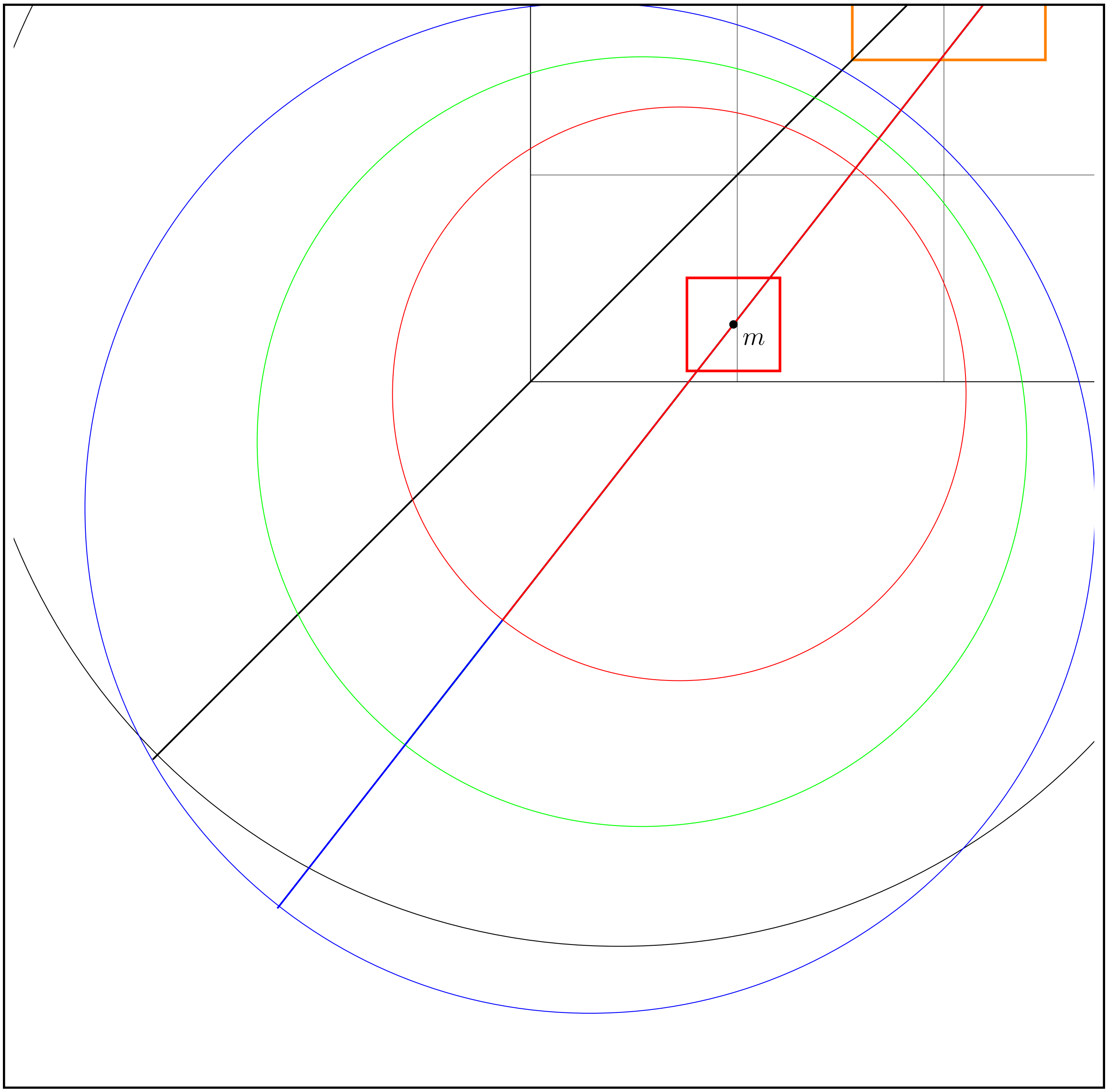

Figure 19:
Apollonian Circle of Earth Venus Mercury

6.3 Earth Venus Mars

Figure 20 shows the Apollonian Circles of Earth-Venus (Figure 16) and Mars-Earth (Figure 18). All three Mars-Earth circles intersect the Apollonian Circle of Earth-Venus. That is, the model of the solar system holds at SMa and either extreme of Mars orbit.

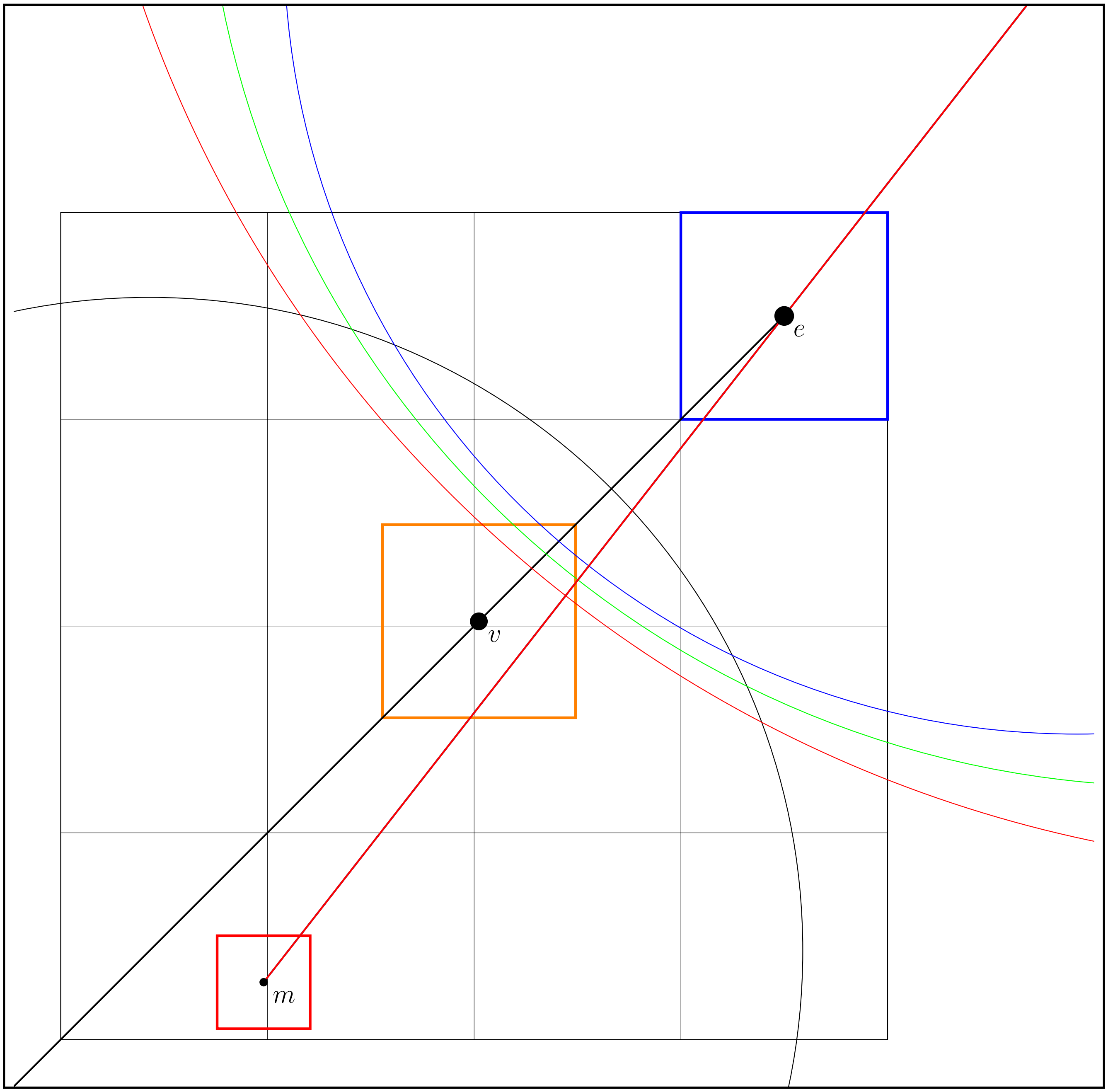

Figure 20:
Apollonian Circles of Earth Venus Mars

7 Conclusions

Mathematics of proportion cannot falsify a general model of the Giza plateau aligned to the inner planets in our solar system. Alignments involving Mercury are meaningless over considerable portions of its eccentric orbit. Alignments involving Mars are meaningful over its entire orbit.